M.Sc. Mathematics - Br. I (A)

under

Choice Based Credit System

2018 - 2020

Syllabi and Regulations

Ramanujan Institute for
Advanced Study in Mathematics

University of Madras

June 2018

1. Structure of the course

M.Sc. Branch I (A) Mathematics (CBCS) 2018 - 2020

Title of the Course		Credit			
	C/E/S	L	T	P	C
SEMESTER - I					
Linear Algebra	C	3	1	0	4
Real Analysis	C	3	1	0	4
Ordinary Differential Equations	C	3	1	0	4
Number Theory	C	3	1	0	4
Elective - 1	E	2	1	0	3
Soft Skill	S				2
SEMESTER - II					
Algebra	C	3	1	0	4
Topology	C	3	1	0	4
Partial Differential Equations	C	3	1	0	4
Computational Mathematical Laboratory	C	3	1	0	4
Elective-2	E	2	1	0	3
Elective-3	E	2	1	0	3
Soft Skill	S				2
SEMESTER - III					
Complex Analysis	C	3	1	0	4
Measure and Integration Theory	C	3	1	0	4
Probability Theory	C	3	1	0	4
Elective - 4	E	2	1	0	3
Elective - 5 / Elective for other Departments*	E	2	1	0	3
Soft Skill	S				2
Summer Internship	S				2
SEMESTER - IV					
Advanced Analysis	C	3	1	0	4
Differential Geometry	C	3	1	0	4
Functional Analysis	C	3	1	0	4
Project / Seminar	C	3	1	0	4
Elective - 6	E	2	1	0	3
Elective - 7	E	2	1	0	3
Soft Skill	S				2

Note: Compulsory Components for Postgraduate Programmes

Core Courses(C)	- 60 Credits
Elective Courses (E)	- 21 Credits
Soft Skill Courses(S)	- 08 Credits
Internship (S)	- 02 Credits
Total	**- 91 Credits**

2. Elective courses offered by RIASM

Title of the Course
Discrete Mathematics*
Number Theory and Cryptography
Programming and Soft Computations*
Difference Equations
Lie Algebras
Stochastic Processes
Representation Theory of Finite Groups
Graph Theory*
Lie Groups of Transformations and Ordinary Differential Equations
Lie Groups of Transformations and Partial Differential Equations
Potential Theory in R^n
Banach Algebras and Operator Theory
Algebraic Number Theory
Mathematical Theory of Electromagnetic Waves
Algebraic Topology
Calculus of Variations and Integral Equations
Fluid Dynamics
Algebraic Geometry

3. Self study Courses

Financial Calculus
Fuzzy Analysis
Harmonic Function Theory
Introduction to Fractals
Lie Groups and Lie Algebras
Probability on Abstract Spaces
Quantum Computations

Master's Programme in Mathematics Syllabi of Courses - Abstract

MSI C001	Linear Algebra	3	1	0	4

Pre-requisite: Undergraduate Linear Algebra

Unit I

Review of Vector spaces - Linear Transformations - Representation of Transformations by Matrices-Linear Functionals - Algebra of Polynomials- Determinants - Properties of determinants-Characteristic Polynomials- Characteristic values - Characteristic vectors - minimal Polynomials.

Unit II

Invariant subspaces - Direct sum Decompositions - Diagonalization of linear operators - Primary Decomposition Theorem.

Unit III

Cyclic Vectors - Cyclic subspaces - Cyclic Decomposition Theorem - Generalised Cayley - Hamilton Theorem- Rational form - Jordan Canonical form.

Unit IV

Bilinear forms - positive - definite, symmetric and Hermitian forms - Sylvester's theorem.

Unit V

Spectral representation of symmetric, Hermitian and normal operators - Applications.

Books for Reference:

1. Kenneth Hoffman and Ray Kunze, Linear Algebra. Prentice Hall of India Private Ltd. New Delhi 2005.
2. Michel Artin, Algebra. Prentice Hall of India Private Ltd. New Delhi 1994.
3. S.H. Friedberg, A.J. Insel, L.E. Spence Linear Algebra, 4th Edition, Pearson, 2002.

MSI C002	Real Analysis	3	1	0	4

Pre-requisite: Undergraduate Level Analysis

Course Objective:

To Provide a systematic development of Riemann -Stieltjes Integral and the calculus on R^n

Unit I

Review : The Real Field - Euclidean Spaces - Metric Spaces - Compactness - Connectedness - Differentiability - Mean Value Theorem - Continuity and Compactness - Continuity and Connectedness.

Unit II

Riemann - Stieltjes Integral : Definition and properties of the integral – Integration and Differentiation - Integration of Vector Valued functions.

Unit III

Sequences and series of functions : Pointwise Convergence - Uniform Convergence - Weierstrass Approximation Theorem - Power Series - Fourier Series.

Unit IV

Functions of Several Variables: Derivatives of a function from R^n to R^m - Chain Rule - Partial Derivatives -Derivatives of Higher order.

Unit V

Basic Theorems of Differential Calculus: Inverse function Theorem -Implicit function Theorem - Rank Theorem.

Books for Reference:

1. Walter Rudin, Principles of Mathematical Analysis, Third Edition, McGraw Hill 1976.
2. N.L. Carothers, Real Analysis, Cambridge University Press, 2000.
3. Tom. M. Apostal, Mathematical Analysis, Narosa, 2000.

MSI C003	Ordinary Differential Equations	3	1	0	4

Pre-requisite : Undergraduate Differential Equations

Unit I

HIGHER ORDER LINEAR EQUATIONS

General Theory of nth order Linear Equations - Homogeneous Equations with Constant Coefficients - The Method of Undetermined Coefficients - The Method of Variation of Parameters.

Unit II

POWER SERIES SOLUTIONS AND SPECIAL FUNCTIONS

Series Solutions of First Order Equations - Second Order Linear Equations - Ordinary Points - Regular Singular Points - Gauss's Hyper-geometric Equation.

Unit III

SOME SPECIAL FUNCTIONS OF MATHEMATICAL PHYSICS AND EXISTENCE AND UNIQUENESS THEOREM

Legendre Differential Equations : Solutions and its Properties - Bessel's Differential equations: Solutions and its Properties - The Method of Successive approximations - Existence and Uniqueness Theorem.

Unit IV :

NONLINEAR DIFFERENTIAL EQUATIONS AND STABILITY

The Phase plane - Linear systems - Autonomous systems and stability - Almost Linear systems- Competing species - Predator - Prey equations - Liapunovs is method.

Books for Reference:

1. George F. Simmons, Differential Equations with applications and historical notes, Tate Mcgraw Hill, New Delhi, 1994.
2. Williams E. Boyce and Richard C. Di Prima, Elementary Differential Equations and boundary value problems, John wiley and Sons, New York, 1967.
3. Earl A. Coddington and Norman Levinson, Theory of Ordinary Differential Equations, Mcgraw Hill, 2017

MSI C004	Number Theory	3	1	0	4

Pre-requisite: UG level algebra and calculus

Unit I

Divisibility and Congruences – Divisibility, primes, Congruences

Unit II

Chinese remainder theorem, Prime power moduli, Primitive roots, Congruences of degree two

Unit III

Quadratic residues, Quadratic reciprocity, Quadratic forms, Sums of two squares

Unit IV

Arithmetic functions, Mobius inversion formula, Linear Diophantine equations, Pythagorean triangles

Books for Reference:

1. Niven, H. S. Zuckerman and H. L. Montgomery, An introduction to the Theory of Numbers, Fifth Edition, John Wiley And Sons, 2000.
2. Victor Soup, A Computational Introduction to Number Theory and Algebra, Cambridge University Press, 2008.
3. Andrew Weil, Number Theory for Beginners, Springer, 1979.
4. David Burton, Elementary Number Theory, 7th Edition Mcgraw Hill, 2012.

MSI C005	Algebra	3	1	0	4

Pre-requisite : Groups, homomorphisms, quotient groups.

Unit I

Symmetry: Groups of motions of the plane, finite groups of motions.
Group actions by left multiplication/conjugation, counting formula.

Unit II

Class equation, symmetric groups, Sylow's theorems.

Unit III

Polynomial rings, Division with remainder, homomorphism and ideals, Properties of ideals, Rings of fractions, Principal Ideal domains, Polynomial rings over fields.

Unit IV
Algebraic and transcendental elements, degree of a field extension, adjunction of roots, splitting fields.

Unit V
Galois Theory: Galois Extensions, Examples.

Books for Reference:

1. M. Artin, Algebra, Pearson publishers, 2ndedition, 2017
2. D. S. Dummit, R. M. Foote, Abstract Algebra, Wiley Student edition- 2nd edition, 2008
3. J. B. Fraleigh, A First Course in Abstract Algebra, Pearson publishers, 7th Edition, 2002.
4. T. W. Hungerford , Abstract Algebra: An Introduction, 3rd Edition Cengage Learning 2012.
5. I.N. Herstein, Topics In Algebra 2 Edition, Wiley India Pvt Ltd, 2006.

MSI C006	Topology	3	1	0	4

Pre-requisite: MSI C002.

Unit I
Topological spaces - subspaces – product spaces – continuous functions - homeomorphisms

Unit II
Connectedness - compactness

Unit III
Separation properties - Urysohn's lemma - Tietze's extension theorem.

Unit IV
Separable and second countable spaces - metrization theorems.

Unit V
Homotopy - fundamental group - induced homomorphisms - covering spaces - fundamental group of the circle.

Books for Reference:

1. James R. Munkres, Topology, 2nd Edition, Pearson Education, 2000.
2. G.F. Simmons, Introduction to Topology and Modern Analysis, 1st Edition, Mcgraw Hill Education, 2004.
3. J.L. Kelly, General Topology, Springer, 2005.
4. James Dugundji, Topology, Allyn and Bacon, 1966.
5. R. Vaidyanathaswamy, Set Topology, Courier Corporation(Dover), 1960.

MSI C007	Partial Differential Equations	3	1	0	4

Pre-requisite: Undergraduate Partial Differential Equations

Unit - I

FIRST ORDER PARTIAL DIFFERENTIAL EQUATIONS : Cauchy's problem for First Order Equations - Linear Equation of the First Order - Integral surfaces Passing through a Given Curve - Surfaces orthogonal to a given system of Surfaces - Compatible system of First Order Equations - Charpits Method.
(**Chapter 2 :** 2.3 – 2.6, 2.9 & 2.10 Ian N Sneddon)

Unit - II

Special Types of First Order Equations - Jacobi's Method.
SECOND ORDER PARTIAL DIFFERENTIAL EQUATIONS : Linear Partial Differential Equations of Second Order with constant coefficients – Classification of Second Order Partial Differential - Canonical Forms - characteristic equations.
(**Chapter 2 : 2.11 & 2.13** Ian N Sneddon & **Chapter 1 :** 1.2 & 1.3 K. Sankara Rao)

Unit - III

ELIPTIC PARTIAL DIFFERENTIAL EQUATIONS : Derivation of the Laplace Partial Differential Equations - Boundary Value Problems – Separation of Variables – Dirichlet Problem for a rectangle - The Neumann Problem for a Rectangle .
PARABOLIC PARTIAL DIFFERENTIAL EQUATIONS : Occurrence of the Diffusion Equation - Boundary Conditions - Elementary Solutions of the Diffusion Equations - Separation of Variables.

HYPERBOLIC PARTIAL DIFFERENTIAL EQUATIONS : Occurrence of the Wave Equations - Derivation of the One Dimensional Wave Equation - Solution of One Dimensional Wave Equation by Canonical Reduction - The Initial Value Problems : D'Alembert's Method.
(**Chapter 2 :** 2.1, 2.2 , 2.5 – 2.7; **Chapter 3**: 3.1 – 3.3, 3.5; **Chapter 4**: 4.1 – 4.4 K. Sankara Rao)

Unit - IV

LAPLACE TRANSFORM METHODS : Solutions of the Diffusion Equations - Solutions of the Wave Equations.
FOURIER TRANSFORM METHODS : Solutions of the Diffusion Equations - Solutions of the Wave Equations - Solutions of the Laplace Equations
(**Chapter 6 :** 6.13 ; **Chapter 7**: 7.11 – 7.13 K. Sankara Rao)

Books for Reference :

1. Ian N Snedon, Elements of Partial Differential Equations, Mcgraw Hill International Book Company, 1957.
2. K. Sankara Rao, Introduction to Partial Differential Equations, Prentice Hall of India, 2007.
3. C.R. Chester, Techniques in Partial Differential Equations, Mcgraw Hill, 1970.

MSI C008	Computational Mathematical Laboratory	3	1	0	4

Prequisite: Calculus, Linear Algebra, basic knowledge of Differential Equations and some knowledge of Programming Language.

Course Objective:
To emphasize the fundamentals of numerical computation and analysis: to explain how, why, and when numerical methods can be expected to work along with soft computational techniques using MAPLE/MATHEMATICA.

Unit - I : Introduction of Maple :
Plotting Curves-Composition of functions, inverses-Sequences and series (finite and infinite sum)-Slope of a line, a secant, a tangent-Equations of tangents-Limit and continuity-2-D and 3-D graphs-Symbolic - Differentiation and Symbolic Integration-Conversion of coordinates, Areas in Polar coordinates Symbolic manipulation on matrices

Unit - II Developing Programs/Procedures:
Linear Algebra, Analysis and Differential Equations

Unit - III
Numerical Analysis :
Non-Linear Equations: Bisection Method, Regula - falsi Method, Newton - Raphson MethodSecant Method,Fixed Point Iteration, System of linear Equations Gauss Elimination, Gauss-Seidel Method, Interpolation, Lagrange's Interpolation Formula, Newton Interpolation, FormulaNumerical Differentiation, Differentiation using limits, Differentiation using Extrapolation, Gauss EliminationGauss-Seidel MethodInterpolation, Lagrange's Interpolation Formula, Newton Interpolation Formula.

Unit - IV
Numerical Differentiation, Differentiation using limits, Differentiation using Extrapolation. Numerical Integration, Composite Tapezoidal Rule, Composite Simpson's 1/3 Rule,, Numerical Solution to Differential Equations, Euler's Method,Taylor's Method of order 4, Runge-Kutta Method of order 4, Milne-Simpson Method.

Books for Reference :
1. R.L. Burden, J. Douglas Faires, Numerical Analysis, Brooks Cole, 9th Edition, 2011.
2. I. Shingareva, C.L. Celeya, Maple and Mathematica, Springer, 2007.

Pre-requisite : Undergraduate Real and Complex Analysis.

MSI C009	Complex Analysis	3	1	0	4

Unit I

A quick review of basic Cauchy Theory: Cauchy's Theorem and Cauchy's integral formula for convex regions, Morera's Theorem, power series representation of analytic functions, zeros of analytic functions, open mapping theorem, argument principle, Rouche's theorem, maximum modulus theorem, Schwarz lemma, Weierstrass theorem on limits of analytic functions.

Unit II

Isolated singularities, Laurent series, Casorati-Weierstrass theorem, meromorphic functions, Mittag-Leffler's theorem, Weierstrass product theorem, gamma function.

Unit III

Homology and homotopy versions of Cauchy's theorem, simply connected regions, normal families, Riemann mapping theorem.

Unit IV

Harmonic functions, mean value property, Poisson integral, Dirichlet problem for the disc, Harnack's inequality. Harnack's principle.

Unit V

Riemann zeta function, functional equation, Euler product, elliptic functions, Weierstrass $\wp$-function.

Books for Reference :

1. John B. Conway, Functions of One Complex Variable I, 2nd Edition, Springer, 1978.
2. Ahlfors, L.V. Complex Analysis, 2nd Ed Mcgraw - Hill, New-york, 1966.
3. Ponnusamy, S., Foundations of Complex Analysis, 2nd Ed. Narosa Publications, 2005.
4. Ponnusamy, S. Silverman, H., Complex Variables waith Applications, Birkhauser, 2006.
5. Stein, E.M., Shakarchi, R., Complex Analysis, Princeton University Press, 2003.

MSI C010	Measure and Integration Theory	3	1	0	4

Pre-requisite: Undergraduate level Real Analysis

Course Objective : To develop the theory of integration via measure, the knowledge of which is essential for working in most branches of modern Analysis.

Unit - I

Lebesgue outer measure, Measurable sets, Regularity, Measurable functions, Borel and Lebesgue Measurability.

Unit - II

Integration of non- negative functions, the general integral, Integration of series, Riemann and Lebesgue integrals.

Unit - III

Differentiation, Integration with respect to a measure, Measure spaces, Completion of a measure, Integration in measure spaces.

Unit - IV

Signed measures and Hahn Decomposition, The Jordan decomposition, Lebesgue Decomposition.

Unit - V

Measurability in a product space, the product measure and Fubini's theorem, Lebesgue measure in Euclidean space.

Books for Reference :

1. G.De Barra, Measure Theory and Integration, New Age international (P) Ltd. 2013.
2. H.L. Royden, Real Analysis, 3rd ed PH 1 Learning Private Ltd, 2010.
3. I.K. Rana, An introduction to measure and Integration, Narosa publishing house, 2005

MSI C011	Probability Theory	3	1	0	4

Pre-requisite: Under Graduate level Calculus

Unit - I : Probability space

Axiomatic definitions for probability space(finite, countably infinite and uncountably infinite outcome spaces)- Events - Fields of events- σ - fields of events - conditional probability and Bayes' theorem

Unit - II : Random Variables and their distributions

Random Variables - distributions function - decomposition of distribution function - probability mass function and Probability density function - Classification of Random Variables - Moments and inequalities - Functions of Random Variables - Discrete and continuous distributions

Unit - III: Independence, conditioning and Convergence

Independence of events - of σ - fields –of Random Variables- conditional expectation - Radon - Nikodym derivatives - convergence of Random Variables(in Prob., a.s., in dist., r-th mean)

Unit - IV: Characteristic functions

Definitions and Simple properties - Inversion theorem - Moments and Characteristic functions - Weak convergence

Unit V : Limit Theorems

Zero-one Laws - WLLN and SLLN for iid and id random variables. CLT for iid and id random variables

Books for Reference :

1. Kai Li Chung, A course in Probability Theory, Academic Pres, 2001.
2. Ash, Probability and Measure Theory, 2nd Edition, Elsevier(Academic Press) 2000.

3. William Feller, An Introduction to Probability: Theory and its Applications, 3rd Ed, Vol 1, Wiley India, 2008.

MSI C013	Advanced Analysis	3	1	0	4

Pre-requisite: MSI C002, C005

Unit - I

Differential forms, integration of forms, Stokes' theorem classical vector analysis.

Unit - II

Ahlfors' Schwarz lemma, Pick's lemma, hyperbolic geometry in the unit disc, Schottky's theorem, the Big Picard theorem.

Unit - III

Analytic continuation, the monodromy theorem, Riemann surfaces, uniformization theorem and covering surfaces.

Unit - IV

Fourier series: Dirichlet's theorem, norm convergence, Fejer's theorem, Riemann-Lebesgue lemma uniqueness theorem, completeness of exponentials in L^2, Parseval formula, isoperimetric inequality, heat equation.

Unit - V

Fourier transforms: Convolution, differentiation and Fourier transform, Schwartz space of rapidly decreasing functions, inversion and plancherel theorems.

Books for Reference :

1. Diff. forms and stokes theorem
   Spivak, Calculus on manifolds, West View Press, 1971.
   Munkres, Analysis on manifolds, Addison Wesley, 1991.
   Manfredo P do carmo, Differential forms and applications, Springer, 1994.

2. Complex Analysis
   Steven G Krantz, Geometric Function Theory, Birkhauser, 2006.
   Raghavan Narasimhan, Complex Analysis in one variable, Birkhauser, 2001.
   T.W. Gamelin, Complex Analysis, Springer, 2001.

3. Fourier Analysis
   Rajendra Bhatia, Fourier Series, Cambridge University Press, 2005.
   Dym and Mckeau, Fourier Series and Fourier Integrals, Academic Press (Revised) edition, 1985.
   Stein and Shakarachi, Fourier Analysis, Pinceton University Press, 2011.

MSI C014	Differential Geometry	3	1	0	4

Pre-requisite: MSI C002 and MSI C006

Course Objective : To give a modern introduction to differential geometry of curves and surfaces.

Unit I

Curves in $R,^3$ Tangent , normal and binormal vectors, curvature and torsion , Plane curves.

Unit II

Smooth surfaces, Examples of Smooth surfaces, tangent and normal vectors, first fundamental form.

Unit III

Differential derivative of vector fields, computation of Christoffel symbols, Length and Area, Isometries.

Unit IV

Weingarten map and the second fundamental form, Gaussian and mean curvatures.

Unit V

Gauss formula, Gauss equation, Codazzi-Mainardi Equations, Theorema Egregium .

Books for Reference :

1. Ethan D. Block, first course in geometric topology and differential geometry, Birkhauser.
2. Andrew Pressley, Elementary differential geometry, Springer Undergraduate Mathematics series, 2010.
3. Manfredo P. do Carmo, Differential Geometry of Curves and Surfaces, Dover Publications; 2 Upd Rev edition (December 14, 2016).

MSI C015	Functional Analysis	3	1	0	4

Pre-requisite: Knowledge of MSI C002 and C005.

Unit - I

Fundamentals of normed spaces, Completeness, continuity of linear maps, Hahn Banach theorems and their applications.

Unit - II

Dual spaces, dual of l^P, L^p , Uniform boundedness principle, closed graph and open mapping theorems

Unit - III

Inner product spaces, orthonormal sets - ,Riesz - Fischer theorems, Riesz Representation theorem.

Unit - IV

Bounded operators and adjoints, Projections, Projection theorem , Normal, Unitary and self-adjoint operators, spectrum of a bounded operator

Unit - V

Compact Operators, Spectral Theorem for Compact Selfadjoint Operators.

Books for Reference :

1. Balmohan V. Limaye, Functional Analysis, New Age International Ltd Publishers, 2017.
2. Simmons, G.F. Introduction to Topology and Modern Analysis, McGraw Hill Education; 1st Edition 2017.
3. George Bachman and Lawrence Narici, Functional Analysis, Dover Publications, 2nd Edition, 1998.

MSI C016	Project / Seminar	3	1	0	4

Course Outline :

- Each student is assigned a topic for term paper and seminar. It is an individual work. During the term, the student will meet periodically the faculty to discuss different stages of the term paper preparation and seminar.

- Preparation of Term Paper : Choosing a Topic in consultation with the Student Advisor - Finding sources of materials - Gathering the relevant findings - Outlining the Paper-j Writing the first draft (manuscript) - Converting into Compuscript (using Latex) and Editing the paper as advised by the Advisor.

- Exposure to collect research papers and to prepare documentation of results in a scientific manner with proper citation principles will form integral part of the course.

- Every student is expected to give at least 3 seminar talks during the course of study. Topics for seminars will be approved by the faculty. Preparation of seminar talks in the form of compuscripts (using mathematical software Latex) is compulsory and talks based on the Term paper be delivered using blackboard/ OHP/ LCD.

- Evaluation is based on the (i) Preparation of the compuscript (30%) (ii) Presentation style (30%) (iii) Oral presentation (40%). Passing minimum 50% of (i), (ii) and (iii) put together.

Books for Reference :

1. English Expression :

1. Carey, G.V. Punctuation, Cambridge University Press.
2. Partridge, E. Usage and Abusuge : A guide to good English, Middlesex, Penguin.

2. Research Writing :

1. Berry, R. : How to write a Research paper. Pergamon Press, London.
2. Cooper, B.C. : Writing Technical Reports, Middlesex , Penguin.

3. Turabian, Kate L. : A manual for writers of term paper, Thesis and Dissertations, University of Chicago Press.

3. Mathematical Typesetting Software:

1.Leslie Lamport . LaTeX : A Documentation Preparation System User's Guide and Reference Manual , Addison Wesley, Mass, 1994.
2.Goossens, Rahtz, and Mittelbach. The LaTeX Graphics Companion, Addison Wesley, Mass, 1997.
3.George Gratzer, First Steps in LaTeX, Birhauser, 1999.
4.George Gratzer, Math Into LaTeX, Birhauser, 2000.
5.F. Mittelbach and M Goossens with Braams, Carlisle, and Rowley, The LaTeX Companion, second edition, Addison Wesley. Mass, 2004.

4. Mathematical Writing :

1.N.E. Steenrod, P.R. Halmos, M.M. Schiffer and J.E. Dieudonne. How to write Mathematics, AMS Publication, 1973.
2.Steven G.Krantz. A Primer of Mathematical Writing, AMS Publication, 1997.
3.Ellen Swanson, Mathematics into Type, (updated Edition) AMS Publication, 1999.
4.Steven G. Krantz. Mathematical Publishing, AMS Publication, 2005.

Syllabi for various Elective Courses

MSI E001	Discrete Mathematics	2	1	0	3

Pre-requisite: Basic algebra

Unit I

Mathematical Logic: Connection - Normal Forms - Theory of Inferences - Predicate Calculus.

Unit II

Set Theory: Operations on Sets - Basic Set Identities - Relations and Orderings.

Unit III

Recursion: Functions - Recursive Functions - Partial Recursive Functions.

Unit IV

Graph Theory: Basic Concepts of Graph Theory - Paths - Connectedness - Matrix Representation of Graphs - Trees - List structures and Graphs

Unit V

Grammars and Languages: Free Semigroups - Grammers and Languages.

Books for Reference :

1. David J. Hunter, Essential of Discrete Mathematics, Jones & Bartlett Learning; 3rd edition, 2015.
2. P. Geetha, Discrete Mathematics, Scitech Publications(India) Pvt. Ltd, 2007.

MSI E002	Number Theory and Cryptography	2	1	0	3

Pre-requisite: Basic algebra and calculus

Unit I

Elementary Number theory

Divisibility and the Euclidean Algorithm, Congruences, Finite fields and Quadratic residues, Cryposystems, Enciphering matrices, Public key Cryptography, RSA, Discrete Log, Knapsack, Primality and Factoring.

Unit II

Introduction to classical cryptosystems

Some simple crypto systems , en ciphering matrices, DES

Unit III

Finate fields and quadratic residues

Finite fields, quadratic residues and reciprocity.

Unit IV

Public Key Cryptography

The idea of a public key Cryptography, RSA, Discrete Log, Algorithms to find discrete logs in finite Fields: Shank's giant – step - baby -step algorithm, Silver-Pohlig – Hellman's algorithm, Diffie - Hellman key - exchange system, ElGamal, zero – knowledge protocols.

Unit V

Primality - Factoring and Elliptic curves.

Pseudoprimes and strong Pseudoprimes, some methods to factor a composite integer:Pollard's rho method, fermat factorization and factor bases, the quadratic Sieve method, elliptic curves-basic facts, elliptic curve cryptosystems.

Books for Reference :

1. Neal Koblitz, A course in Number Theory and Cryptography, Springer, Second Edition, 1994.
2. James S. Kraft, An Introduction to Number Theory with Cryptography, CRC Press, 2013.

MSI E003	Programming and Soft Computations	2	1	0	3

Unit I

Tokens, Expressions and Control Structures Functions in C++

Unit II

Classes and Objects - Constructors and Destructors

Unit III

Operator Overloading and Type conversions - Inheritance

Unit IV

Pointers - Virtual Functions and Polymorphism - Templates and Exception handling

Unit V

Maple/Mathematica Commands (without programming)

Books for Reference :

1. Subrata Saha, Basic Computation and Programming with C, Cambridge University Press; 1st Edition, 2017

MSI E005	Lie Algebras	2	1	0	3

Pre-requisite: Knowledge of MSI C001 and C004

Unit I

Basic Concepts of Lie Algebras

Unit II

Ideals and homomorphisms

Unit III

Solvable and nilpotent Lie algebras

Unit IV

Semisimple Lie algebras : Theorems of Lie and Cartan, Killing form

Unit V

Complete reducibility of representations and representation of sl(2,F).

Books for Reference :

1. Georgia Benkart and J.Marshall Osborn, Lie Algebras, Springer, 1989.
2. J.E. Humphreys, Introduction to Lie Algebras and Representation Theory, Springer, 1997.

MSI E006	Stochastic Processes	2	1	0	3

Pre-requisite: MSI C010

Unit I

Markov chains, an introduction- Definitions, Transition probability matrix of a Markov chain, some Markov chain models, First Step Analysis, some special Markov chains, Functionals of Random Walks and Success runs

Unit II

Long run behaviour of Markov chains - Regular Markov chains - Transition probability matrices - Examples, Classification of states, Basic limit theorem of Markov chains, Reducible Markov chains

Unit III

Poisson Processes - Poisson distribution and Poisson Processes, Law of rare events, distributions associated with Poisson Processes, Uniform Distribution and Poisson Processes, Spatial Poisson Processes, Compound and Marked Poisson Processe

Unit IV

Continuous time Markov chains - Pure birth processes – Pure Death processes, Limiting behaviour of birth and death Processes, birth and death Processes with absorbing states Finite state Continuous time Markov chains, A Poisson Process with a Markov intensity

Unit V

Renewal phenomena - Definitions, examples, the Poisson Process viewed as a renewal process

Books for Reference :

1. J. Medhi, Stochastic Processes, 2nd Edition, New Age International (P) Limited, 1984.
2. S. Karlin and H.M. Taylor, Stochastic Processes, Academic Press, 2nd Edition, 1981.

MSI E007	Representation Theory of Finite Groups	2	1	0	3

Pre-requisite: MSI C001 and C004

Unit I

Classical groups: General linear group , Orthogonal group, Symplectic group, Unitary group.

Unit II

Group representation, conjugate representation, G-invariant spaces - irreducible representations - Schur's lemma

Unit III

The Group Algebra - Maschke's theorem - characters. Orthogonality relations for characters – Number of irreducible representations

Unit IV

Permutation representations - Regular representation. Representations of Symmetric groups

Unit V

Representation of Finite abelian groups - Dihedral groups.

Books for Reference :

1. Serre, Jean-Pierre, Linear Representations of Finite Groups, Springer verlag. (1977),
2. Bruce sagan The Symmetric Group, Representations, Combinatorial Algorithms, and Symmetric Functions

MSI E008	Graph Theory	2	1	0	3

Pre-requisite: Undergraduate Algebra

Unit I

Graphs - Vertex degrees - Sub-graphs - Paths and cycles - Connected graphs - Connected components

Unit II

A cyclic graphs - Trees - Cut edges - Cut vertices - Spanning Tree .

Unit III

Euler tours - Euler graphs - Hamiltonian paths - Hamiltonian graphs - Closure of a graph.

Unit IV

Planar graphs - Euler's formula- Vertex colouring - Chromatic number - Chromatic polynomial - R - Critical graphs.

Unit V

Edge colouring - Edge Chromatic number - Dual of a plane graph - Map colouring - Four and five colour theorems.

Books for Reference :

1. John Adrian Bondy and U.S.R. Murty, Graph Theory With Applications, Elsevier Science Ltd/North-Holland, 1976.
2. Douglas West, Introduction to Graph Theory (2^{nd} Edition), Pearson, 2017.

MSI E009	Lie Groups of Transformations and Ordinary Differential Equations	2	1	0	3

Pre-requisite: MSI C002 and C005

Unit I

Introduction - Lie groups – elementary examples - Lie groups of transformations - Infinitesimal transformations - First Fundamental Theorem of Lie - Canonical coordinates.

Unit II

Extended group transformations and infinitesimal transformations (one independent and one dependent variables) Multi - Parameter Lie groups of transformations second and Third Fundamental Lie's Theorem

Unit III

Lie Algebras - elementary examples - classification of sub algebras - Applications.

Unit IV

Invariance of first and second order ordinary differential equations. Applications of Lie groups of transformations to ordinary differential equations

Books for Reference :

1. G.W. Bluman and S. Anco, Symmetry and Integration for Differential Equations, Springer, 2002.
2. P.J. Olver, Applications of Lie groups to Differential Equations, Springer (1998) Berlin.

MSI E010	Lie Groups of Transformations and Partial Differential Equations	2	1	0	3

Pre-requisite: MSI E009

Unit I

Introduction - Review of first and second order partial differential equations with two independent variables.

Unit II

Extended group transformations and infinitesimal transformations. with two independent and one dependent variables

Unit III

Invariance of a partial differential equations of second order - Heat equation, one dimensional wave equation, nonlinear Heat equations - Reduction of independent variables - similarity solutions.

Unit IV

Applications of Lie groups of transformations: Kortedweg de Vries equation and nonlinear Heat equations.

Books for Reference :

1. G.W. Bluman and S. Anco, Symmetry and Integration for Differential Equations, Springer, (Berlin)2002.
2. P.J. Olver, Applications of Lie groups to Differential equations, Springer (1998) Berlin.

MSI E011	Potential Theory in R^n	2	1	0	3

Pre-requisite: MSI C009

Unit I

Harmonic functions - Dirichlet problem.

Unit II

Functions harmonic on a ball - Directed families of harmonic functions.

Unit III

Super harmonics functions – Equivalent definitions - Minimum principle.

Unit IV

Properties of Super harmonic functions

Unit V

Directed families of super harmonic functions – Properties of surface and volume mean values.

Books for Reference :

1. L.L. Helms, Introduction to Potential Theory John Wiley & Sons Inc; 1st Edition, First Printing edition, (January 1, 1969).
2. Bengt O. Turesson, Nonlinear Potential Theory and Weighted Sobolev Spaces, Springer, 2000.
3. Oliver Dimon Kellogg, Foundations of Potential Theory, Springer, 2012.

MSI E013	Banach Algebras and Operator Theory	2	1	0	3

Unit I

Banach Algebras definition, examples, ideals and quotients, invertibility and the Spectrum, Banach – Mazur theorem.

Unit II

Spectral radius formula, Gelfand theory of commutative Banach Algebras.

Unit III

C* - Algebras, Selfadjoint, normal, unitary operators on a Hilbert space, Projections.

Unit IV

Gelfand – Naimark Theorem for commutative C* - algebras, continuous functional Calculus for normal operators, Positive Operators and Square root.

Books for Reference :

1. George Finlay Simmons, Introduction Topology And Modern Analysis, TBS; 1st Edition 2003.

MSI E014	Algebraic Number Theory	2	1	0	3

Pre-requisite: MSI C001 and C005

Unit I

Algebraic Background – Symmetric Polynomials - Modules - Free Abelian Groups

Unit II

Algebraic Numbers - Conjugates and Discriminants - Algebraic integers - Integral Bases - Norms and Traces - Rings of Integers - Noetherian rings and Noetherian Modules.

Unit III

Quadratic fields and Cyclotomic fields - and integers in Quadratic fields and Cyclotomic fields

Unit IV

The group of units - The factorization into irreducible elements - examples of non-unique factorization into irreducibles - Euclidean Quadratic fields.

Unit V

Prime factorization of ideals - Dedekind rings- the norm of an ideal - class groups

Books for Reference :

1. Abstarct algebra 3rd edition david s dummit, Richard Foote, 2011, Wiley student edition.
2. Algebraic theory of numbers, Pierre Samuel, Dover books on mathematics, 2013.

MSI E015	Mathematical Theory of Electromagnetic Waves	2	1	0	3

Pre - requisite: Vector Calculus, Real Analysis, Differential Equations.

Course Contents:

Unit - I: Review of Vector Calculus: Space Curves and Surfaces- Gradient, Divergence, Curl-Green's Theorem -Gauss Divergence Theorem.

Unit - II: Electromagnetic Fields: Maxwell's Equations-Electromagnetic Waves-Reduced Wave Equation.

Unit - III: Solutions in Bounded Domain: Fundamental Solutions of Reduced Wave equation - Green's Function-Structure of Wave functions-Representation of Wave functions.

Unit - IV: Solutions in the Exterior Domain: Structure of Wave functions- Sommerfeld's Radiation Conditions- Green's Representation Theorem-Far Field Patterns.

Unit - V: Boundary Value Problems: Boundary Value Problems in the bounded domain-Boundary Value Problems in the Exterior domain-Scattering and Inverse Scattering.

Books for Reference :

1. Foundations of Mathematical Theory of Electromagnetic Waves, Claws Miller Springer

MSI E016	Algebraic Topology	2	1	0	3

Pre - requisite: Algebra (MSI C005), Topology (MSI C006)

UNIT I

Contractible spaces, homotopy, homotopy of paths, fundamental group, induced homomorphisms.

UNIT II

Covering spaces, path lifting lemma, lifting correspondence, fundamental group of the circle.

UNIT III

Retractions, deformation retract, nullhomotopy, fundamental theorem of algebra, homotopy equivalence.

UNIT IV

Simplicial complexes, subdivision, barycentric subdivision, simplical approximation theorem.

UNIT V

Fundamental group of a simplicial complex, Orientation of a simplicial complex, Homology groups, Simplicial homology, homotopy invariance of simplicial homology.

Refrences:

1. Munkres, Topology, Pearson, 2015.
2. Singer and Thrope, Lecture Notes in Elementary Topology and Geometry, Springer, 1967.
3. Satya Deo, Algebraic Topology, Hindustan Book Agency (trim series), 2006.
4. Anant R. Shastri, Basic Algebraic Topology, CRC Press, 2003.

MSI E017	Calculus of Variations and Integral Equations	2	1	0	3

Pre-requisite : Differential Equations.

Unit I: Variations of a functional -Euler-Lagrange Equation - Necessary and sufficient conditions for extrema.

Unit II : Variational Methods for boundary value problems in Ordinary and Partial differential equations.

Unit III : Linear Integral equations of the first and second kind of Fredholm and Volterra type - Solutions with seperable kinds.

Unit IV: Characteristic numbers and eigen functions, resolvent kernal.

Unit V : Symmetric Kernals and Singular integral equations

Books for Reference Books :

1. L. Elsgolts, Differential Equations and the Calculus of Variations Mir Publishers, Moscow, 1973
2. A.S. Gupta, Calculus of Variations with Applications Prentice. Hall of India, New Delhi, 1997.
3. Ram P. Kanwal, Linear Integral Equations. Academic Press, New York, 1971.

MSI E018	Difference Equations	2	1	0	3

Pre - requisite : Differential Equations.

Unit I

Difference Calculus - First order equations - General results for linear equation - Solving linear equation.

Unit II

Equations with variable coefficients - Nonlinear equations that can be linearized.

Unit III

z - transform - solving linear difference equation using z - transform.

Unit IV

Second order linear equation - Sturmian Theory - Green's function.

Unit V

Disconjugacy - Riccati equation - Oscillation.

Books for Reference :

1. W.G. Kelly and A.C. Petersan, Difference Equations: An introduction with Applications Acad. New York,1991
2. S. Elayadi, An Introduction to Difference Equations, Springer New York 2005.
3. R.P. Agarwal Difference Equations and Inequalities, Marsel Dekker, New York, 2000.

MSI E0	Fluid Dynamics	2	1	0	3

Pre - Requisite : Undergraduate Differential Equations and Mechanics.

UNIT-I : Kinematics of Fluids in motion. Real fluids **and** Ideal fluids- Velocity of a fluid at a point, Stream lines , path lines , steady and unsteady flows- Velocity potential - The vorticity vector- Local and particle rates of changes - Equations of continuity - Worked examples - Acceleration of a fluid - Conditions at a rigid boundary.
Chapter 2. Sec 2.1 to 2.10

UNIT-II: Equations of motion of a fluid : Pressure at a point in a fluid at rest.- Pressure at a point in a moving fluid - Conditions at a boundary of two inviscid immiscible fluids- Euler's equation of motion - Discussion of the case of steady motion under conservative body forces.
Chapter 3. Sec 3.1 to 3.7

UNIT-III : Some three dimensional flows. Introduction- Sources, sinks and doublets - Images in a rigid infinite plane - Axi symmetric flows - Stoke's stream function
Chapter 4 Sec 4.1, 4.2, 4.3, 4.5.

UNIT-IV : Some two dimensional flows : Meaning of two dimensional flow - Use of Cylindrical polar coordinate - The stream function - The complex potential for two dimensional , irrotational in compressible flow - Complex velocity potentials for standard two dimensional flows - Some worked examples - Two dimensional Image systems - The Milne Thompson circle Theorem.
Chapter 5. Sec 5.1 to 5.8

UNIT-V: Viscous flows: Stress components in a real fluid. - Relations between Cartesian components of stress- Translational motion of fluid elements - The rate of strain quadric and principle stresses - Some further properties of the rate of strain quadric - Stress analysis in fluid motion - Relation between stress and rate of strain- The coefficient of viscosity and Laminar flow - The Navier – Stoke's equations of motion of a Viscous fluid.
Chapter 8. Sec 8.1 to 8.9

Books for Reference :

1. R.W. Fox and A.T. McDonald. Introduction to Fluid Mechanics, Wiley, 1985.
2. E. Krause, Fluid Mechanics with Problems and Solutions, Springer, 2005.
3. B.S. Massey, J.W. Smith and A.J.W. Smith, Mechanics of Fluids, Taylor and Francis, New York, 2005
4. P. Orlandi, Fluid Flow Phenomena, Kluwer, New Yor, 2002.
5. T. Petrila, Basics of Fluid Mechanics and Introduction to Computational Fluid Dynamics, Springer, berlin, 2004.
6. F. Chorlton, Fluid Dynamics , CBS Publications. Delhi ,1985

MSI E0	Algebraic Geometry	2	1	0	3

Pre-requisite:

Unit I
Rings, polynomial rings, quotient rings, ideals, rings of fractions, principal ideals domain, unique factorisation domain, irreducibility of the polynomials

Unit II
Modules homomorphism, quotient modules, finite modules, free modules, finitely generated modules.

Unit III
Noetherian Rings, Hilbert's basis theorem, k-algebras, affine algebraic sets, coordinate ring of an affine algebraic sets, morphism between affine algebraic sets.

Unit IV
Radicals affine varieties, Zariski topology, primary ideals, primary decomposition, Hilbert's Nullstellensatz theorem.

Unit V

Localisation, Spectrum of rings, local rings of affine algebraic varieties, tangent space of an affine variety, smooth varieties, affine curves

Books for Reference :

1. C. Musili, Rings and Modules, Narosa Publishing House, 1994.
2. Robin Hartshorne, Algebraic Geometry, Springer, 2013.
3. David Dummit & Richard Foote, Abstract Algebra, Wiley, 2011.
4. M. Atiyah, I.Macdonald, Commutative algebra, Hachette UK, 1994.
5. D. Eisenbud, Commutative algebra, with a view toward algebraic geometry, 2013.
6. Algebraic geometry : a first course J. Harris, Springer, 1995.

Self-Study Courses for the Ramanujan Institute only

The detailed syllabi will be provided at the time of registration by the faculty concerned.

MSI S002	Algebraic Topology	SS	0	3	0	3
MSI S003	Financial Calculus	SS	0	3	0	3
MSI S004	Fuzzy Analysis	SS	0	3	0	3
MSI S005	Harmonic Function Theory	SS	0	3	0	3
MSI S007	Introduction to Fractals	SS	0	3	0	3
MSI S008	Lie Groups and Lie Algebras	SS	0	3	0	3
MSI S009	Probability on Abstract Spaces	SS	0	3	0	3
MSI S010	Quantum Computations	SS	0	3	0	3
MSI S011	Quantum Groups	SS	0	3	0	3